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Mathematics in economics: Schmoller, Menger and Jevons

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Abstract *Investigates the economic methodologies of Carl Menger, William Stanley Jevons and Gustav Schmoller with respect to the issue of whether mathematics is or is not an adequate language to express economic relationships. First, Menger's and Jevons's respective methodologies are identified as Aristotelian which means,* inter alia, *that economic properties are real, are naturally related to each other, exist as part of the observable world and can be separated (in thought or otherwise) from other properties. Second, it is shown how this general Aristotelian outlook has very different implications for Menger's and Jevons's thinking about mathematics. Third, these two "monogenetic" views are contrasted with Gustav Schmoller's "polygenetic" approach which holds that a purely deductive economics, based on a small number of self-evident principles, is inadequate for social purposes.*

> *Mathematics becomes very odd when you apply it to people. One plus one can add up to so many different sums . . .*
> (Frayn, 1998, p. 29)

Introduction

The motivation behind this paper stems from my interest in the methodology of contemporary economics, and in particular in defences of its mathematical formalism. Skimming through an up-to-date economics journal, a reader unfamiliar with the subject could easily be convinced that he has got a piece of applied mathematics in front of him. But this has not always been the case. According to Backhouse (1998), the proportion of articles in the *EJ* and *AER* that use algebra rose from only 10 per cent in 1930 to around 75 per cent in 1980. This indicates a great shift in the style of argument with as great an implication for the development of the subject.

The origins of mathematical economics clearly date back at least to the last third of the nineteenth century, with W.S. Jevons, Léon Walras and Irving Fisher among its pioneers[1]. In particular the 1870s and 1880s were a time of intensive discussion about the correct way to go about economics, especially whether mathematics is or is not the right language for an economics argument. The main writings of the protagonists of my case study fall as well

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in this period. I chose Schmoller, Menger and Jevons for my investigation because all three have pronounced opinions on whether maths is or is not adequate: Jevons is an ardent defender, thinking that one not only can apply maths but one has to – the subject matter demands it; Menger's ideal is what he calls "exact" science but that does not mean mathematical science; and Schmoller is well known for his polemics against any attempt to make economics deductive.

In this paper I focus on the more methodological works and comments of the three philosopher-scientists rather than their substantive writings. That is, I look mainly at their arguments about what an ideal economic science has to be like, and especially what place there is for mathematics, and not whether they come close to that ideal in their respective economics. Ultimately, my interest is whether their arguments can help us with contemporary methodological concerns. Here, however, my focus will be historical-descriptive rather than methodological-analytic.

There are three broad lines of thought about the relation between mathematics and the empirical sciences: Platonism or "abstract objectivism", Aristotelianism or "concrete objectivism" and Kantianism or "subjectivism". Very roughly speaking, abstract objectivism is the doctrine that the world is essentially mathematical, phenomena which are not susceptible to exact mathematical description have less reality than those which do. Concrete objectivism locates mathematical structures in perceptible objects. They have to be revealed by abstracting and idealising, that is, by stripping off the non-mathematical features from the mathematical ones. Subjectivism does not think that the world itself is mathematical at all. Quantitative concepts represent qualitative features of the world. But if we are lucky, our concepts may pick out features which bear natural relations among them, and our mathematical laws can adequately describe these relations.

The main thesis of this paper is that both Menger and Jevons are Aristotelians or concrete objectivists but two very different exemplars of the same breed. The next section seeks to defend that thesis. The section after that lumps the two views together as monogenetic and contrasts this with Schmoller's polygenetic methodology. The last section draws some conclusions.

Menger and Jevons: two unlike Aristotelians

Before examining Menger's and Jevons's economic methodologies in more detail, I want to outline what I mean when I call an economist an Aristotelian or a concrete objectivist[2]. The main point is, I think, what I sketched above, *viz.* that the general aspect of an individual thing (for instance, the mathematical aspect) is in the object itself rather than in some abstract realm of ideal forms or in the observing subject. A healthy man is not somebody who takes part in some way or other in the idea of health, but a man who is healthy; "health" exists but it exists because there are healthy people. The same is true for mathematical objects: "three" is not an invention of the human mind nor a Platonic form. "Three" exists in the objects and on account of them, because there are three dates, three children and three palm trees.

As a consequence, science involves a great deal of abstraction, approximation and idealisation, where the emphasis is on the "-isation" rather than the "ideal". The idea is that we treat a concrete object *qua* the aspect we are interested in. For instance, we have a fence of given length and want to put it round an as large as possible plot of land. In this case we treat the fence *qua* geometrical line and geometry tells us that a circle encloses the largest area for a fixed-length perimeter. But that means that we have to abstract from many features of the real fence, for instance its physical features or the aesthetic features it might have. In some cases we will also have to idealise and approximate because the plot of land is hilly but we treat it as a plane in our geometrical exercise, or we have to approximate an ideal circle because the fence is not perfectly flexible.

A corollary to this general idea is that sciences are generally treated as "separate", that is, it is presupposed that it is possible to attend to some features of a situation to the exclusion of others and the abstracted features do not interact with the ones focused on. Our fence will have a certain colour. But this fact is completely irrelevant for the determination of the shape of the plot of land we are going to border. Other features of concrete realism include an ontology of "natural kinds", a robust scientific realism and some version of essentialism. But I will not say much about these because they are not central to my argument. The following list summarises the features of concrete realism I am interested in:

- The general aspect of a thing is in the individual thing itself and neither in some abstract realm of ideas nor in the observer.
- When an object or a situation falls under a specific science, we treat the object or situation *qua* the properties characteristic of that science.
- It makes sense to attend to some aspects of a given object or situation to the exclusion of others. The sciences are *separate*.
- An Aristotelian science will typically involve a great deal of isolation, idealisation and approximation.
- There are in the world certain "essences" or "natures" governed by laws and structures.
- The world is knowable. Science aims at true theories and the objects it posits exist.

The task at hand now consists in first showing that both Menger and Jevons are concrete realists or Aristotelians according to my scheme, and then marking out differences between them, especially with regards to the role of mathematics.

That Menger was an Aristotelian of sorts has been claimed time and again, although his methodology of economics has also been analysed in terms of

other philosophers' ideas, including Plato, Leibniz and Popper[3]. Thus there is not much novelty in my claim but I will stress particular aspects of Aristotelianism, aspects I label "concrete realism" and also highlight features that will contrast nicely with Jevons's rather different type of the same basic idea.

Carl Menger: the complex of Bedürfnisse

Menger starts his methodological essay with a classification of the sciences into three groups, historico-statistical, theoretical and applied. The species of theoretical science has two sub-species, the realistic-empirical and the exact orientation. I am only concerned with exact science here. The exact approach has two aims, these being to find:

(1) "strict" types; and

(2) their "exact" laws or "laws of nature".

"Types" are basically kinds of phenomena. They are "strict" if they are exactly alike, that is, they are instantiations of a universal[4]. "Laws" are relations of coexistence and of succession between types. They are "exact" if they hold universally and by necessity.

How does theoretical science arrive at these aims? Strict types are, for Menger, the simplest elements of a situation: "[Theoretical science] seeks to discover the simplest elements of every real thing, elements which, just because they are the simplest, must be thought of as strictly typical" (Menger, 1883, p. 41)[5]. Menger deals with the second aim immediately after this: "[Theoretical science] examines ... how from the above mentioned simplest, partly almost unempirical, elements of the real world in their (likewise unempirical) isolation from all other influences more complicated phenomena arise ..." (Menger, 1883, pp. 41-2). The basic idea is an attempt to get around the problem of induction: inductive empirical methods can only result in real types and empirical laws, the former of which are only more or less similar but never exactly alike, and the latter hold at best only for the most part and even that is never guaranteed. By attending to the simplest elements and the relations that arise from them, we ascertain strict universality and necessity[6]. The reason is the following:

> [Theoretical science,] however, starts from these presumptions [that the simplest elements can be displayed and that they have a perfectly exact measure] because it would otherwise not be able to achieve the goal of exact research, *viz.*, the discovery of exact laws, whereas, under the assumption of strictly typical elements, of an exact measure of these and their complete isolation from all other causal factors, though, and on the basis of the rule of cognition that was sketched above, it results in laws of the phenomena that are not only exceptionless but they cannot be thought otherwise according to our laws of thought ... (Menger, 1883, p. 42).

The law of thought Menger is referring to is the law of causation: if an A-event is followed by a B-event in a single case, A-events will always, and by necessity, be followed by B-events under the same conditions (Menger, 1883, p. 39f). These ideas can nicely be interpreted along Aristotelian lines. The crucial word in the quote is "isolation": being confronted with a concrete situation, we isolate its simplest elements from all other causal factors and we are thus able to arrive at necessary laws connecting the different ideal elements. Menger tells us that exact science does not consider whether these ideal elements can exist in their isolation in reality. As a consequence, it is a fallacy to believe that exact laws can be tested against "full empirical reality". The relation between the results of exact science and the real world is of the same kind as the relation between geometry and the real world in Aristotle's philosophy of mathematics. Just as there are no perfect cuboids or circles or extensionless points in the real world, and in that sense these things exist partly only in our ideas, so Menger's simplest elements may not be able to exist in isolation (Menger's examples include absolutely pure oxygen, pure alcohol, pure gold and an individual who pursues only economical goals). However, the simplest elements are real features of real situations and produce real causal effects.

The next step is to have a criterion to mark out what the simplest elements of a given situation are. This can be labelled as the "problem of constitution": for a given science, what are the objects falling under that science? A coin, for instance, has a large number of properties. It is made of copper, has a certain colour, a certain weight, a certain size, and a certain purchasing power. If I want to know how many coins I have to line up to get from my desk to the door of my office, I will treat them as objects of geometry. If I want to melt it, it is an object of physics. And if I want to buy things, it falls under the science of economics. Menger's economics falls almost completely out of his definition of the subject matter. He writes, "Under economy [*Wirthschaft*] we understand the precautionary activity of humans that is aimed at the satisfaction of needs of goods, under political economy [*Volkswirthschaft*] the social form of the latter" (Menger, 1883, p. 44). Again, an apple has many properties. But in economics we are interested in human needs. Thus we treat the apple *qua* its capacity to satisfy needs. Everyone who has read Menger's *Principles* will see exactly what I mean. Things that can be brought in a causal connection with the satisfaction of a human need are useful things. When we know about the causal connection, and have power over the thing, it is a good. The good is economic in case it is scarce. Higher-order goods are those with a capacity to produce lower-order goods. And so on.

Thus economics is a separate science. It treats objects *qua* their capacity to satisfy needs and isolates human activities directed at need satisfaction from all other activities. Menger thinks that such a separation is possible. Hence Menger fulfils the first four criteria from my list. The remaining ones follow easily. Menger's exact or strict types are certainly natural kinds[7]. His science aims at true classifications of phenomena and their relations[8]. According to one version of essentialism, it is a necessary truth that if anything possesses a property x essentially, everything which possesses that property x possesses it essentially. But this is exactly what Menger is after: everything which is a good (because it has a good's properties) has its properties by necessity.

What, then, about mathematics? Time and again, Menger emphasises that exact science investigates the nature of strict types and their relations as well as their (exact) measure (Menger, 1883, pp. 40-2). That to me sounds pretty much like quantitative relationships, and these are frequently represented by symbolic formulæ. However, we do not find any more mathematics in the *Principles* than a few numerical examples illustrating the law of diminishing degrees of satisfaction. Menger's son Karl explains this simply by the lack of sufficient mathematical skills:

> It is known that in contrast to Walras and Pareto ... and to Jevons ... the Austrians came to economic theory from jurisprudence, the government, and economic activities, and in the old Austrian Gymnasien, they had not received any instruction in mathematical analysis ... But I am afraid that he [Menger] did not acquire an operative knowledge, let alone a critical insight into calculus (Menger, 1973, p. 44).

Menger himself states his opinion in a series of letters to Walras (Jaffé, 1965, Vol. I, letter 566 and Vol. II, letter 602). The statements made in that correspondence can best be interpreted as an appeal to the division of labour between different branches of economics. Menger sees himself as providing the foundations for his science: investigation about the natures or essences of the things in the economic realm and their relations. At a later, and certainly subordinate, stage, mathematics may be used as a tool, a supplementary science, a method of demonstration or exposition.

This cannot be the whole story, however: why can't we, after "understanding" the thing in question, i.e. grasping its essence, use mathematics in order to determine the quantitative relations between exact types? The answer must lie in what Menger takes the nature of the most basic economic concept to be, that is, the nature of the Mengerian need. I cannot give a whole account of his theory of value here, but I can list some features that make the application of mathematics difficult to say the least: hierarchical ordering of the needs, possible complementarity of goods, many-to-many relationship between goods and needs, absence of substitutability of goods across the whole set, etc. The main point is, I think, that there is no common denominator between the different needs, they cannot be measured on the same unit. Hence Alter (1990) concludes: "Value is purely psychological and the complex of *Bedürfnisse* is lexicographically ordered, implying that even though each value can be represented by a number, a single-valued, continuous 'value function' defined over the domain of all *Bedürfnisse* does not exist." I agree wholeheartedly.

William Stanley Jevons: calculus of pleasure and pain

Before getting back to Menger for a comparison with Jevons, I shall outline Jevons's philosophy of economics and identify Aristotelian traits. Jevons calls his *Theory of Political Economy* an essay about the "mechanics of self-interest and utility" (Jevons, 1871, p. xviii). The concept of utility plays the same role in Jevons's theory as "needs" in Menger's[9]. Utility for Jevons, in good Benthamite tradition, is related to pleasure and pain. Although Jevons inherits the basic notion, he reduces Bentham's seven "circumstances" of pleasure and pain[10] to the two in his view most fundamental ones – "intensity" and "duration". As pain is defined as the negative of pleasure, both can be measured along the same two dimensions.

From its outset, Jevons's theory is very intimately and naturally connected to mathematics. Simply because economics deals with quantities (i.e. qualities capable of intensities), mathematical formulæ are the natural objects to represent economic relations:

> As the complete theory of almost every other science involves the use of that [i.e. differential] calculus, so we cannot have a true theory of Economics without its aid. To me it seems that our science must be mathematical simply because it deals with quantities. Wherever the things treated are capable of being greater or less, there the laws and relations must be mathematical in nature (Jevons, 1871, pp. 3-4).

This defence of the mathematical method is implausible. Many concepts (or things) are capable of intensities without mathematics being the natural language to express their relations. I love my younger brother more than my older brother. Switzerland is more democratic than Nazi Germany. There is more witchcraft to be found among the Azande than among the French. Love, democracy and witchcraft are capable of "being greater or less" but we will need good arguments to put psychology, political history and anthropology methodologically on par with mechanics. That things come in more or less is not sufficient in order to represent it by a mathematical symbol. Nor is it necessary. Colour, not its brightness but the subjective feeling, is a quality rather than a quantity. Blood is not redder than brick but differently red. However, it is possible to produce any colour by different mixtures of light of the three basic colours, and thus we can represent it by a three-placed vector. Whether we can represent something mathematically depends less on whether it comes in intensities than on how it behaves and what we can do with it. Furthermore, we need arguments as to what type of number we should take to represent our "thing": an integer, a real number, a complex number, a vector? Jevons adds a few thoughts about the branch of maths he thinks suits economics.

Through his re-interpretation of utility (that is, the reduction to two dimensions), Jevons makes differential calculus applicable to the subject: both time (duration) and intensity are continuous quantities; pain just is negative pleasure. Feeling now can be represented in a Cartesian co-ordinate system, with intensity on the ordinate and duration on the abscissa. Utility is represented in a co-ordinate system with, again, intensity on the ordinate and quantity of the commodity on the abscissa. Utility u is a function of the commodity x in this graph, and what Jevons calls "degree of utility" is the slope of that function, the first derivative du/dx.

How does Jevons defend this version of marginalism? Two quantities are involved here, and both are supposed to vary continuously[11]. He argues for the continuous variation of intensity only very cursory[12]:

> But it is an artificial assumption that the intensity would vary by sudden steps and at regular intervals ... To avoid error, we must imagine the intervals of time to be infinitely short; that is, we must treat the intensity as varying continuously (Jevons, 1871, p. 33).

The defence of taking the commodity as varying continuously is more interesting from our perspective. The basic idea is that if we take large numbers of consumers rather than one individual, the notion of an infinitessimal change does make sense:

> The notion of infinitely small quantities of food may seem absurd as regards the consumption of one individual; but, when we consider the consumption of a nation as a whole, the consumption may well be conceived to increase or diminish by quantities which are, practically speaking, infinitely small compared with the whole consumption. The laws which we are about to trace out are to be conceived as theoretically true of the individual; they can only be practically verified as regards the aggregate ... (Jevons, 1871, p. 52).

I want to interpret these remarks in the following way. The laws of economics are true of every being that is subject to pleasure and pain and is capable of rational deliberation. However, in the context of individual decisions they may be overshadowed by the imperfections of concrete objects, for instance that I can only buy a discrete amount of cars, and even if my exchange equation tells me to swap 17.2 Koala bear furs for π cars, I have to contend with the ratio 17:3. On the other hand, at the aggregate level one can approximate π to a high degree of accuracy and in this way the laws of economics reveal their operation. Throughout Jevons's work, methodological as well as substantive, the notions of average and approximation play an important role. And I think it is possible to interpret his ideas along realist lines, that is, an average (approximately) measures real causal tendencies. The causal factors we are interested in are often counteracted by disturbances. However, disturbances cancel each other out in the long run and the real tendency can be extracted. Here is what he says in the opening passages of the *Theory of Political Economy*:

> The use of an average, or, what is the same, an aggregate result, depends upon the high probability that accidental and disturbing causes will operate, in the long run, as often in one direction as the other, so as to neutralise each other. Provided that we have a sufficient number of independent cases, we may then detect the effect of any tendency, however slight (Jevons, 1871, p. 17).

Emphasis must be laid on the distinction here made between accidental or disturbing causes and tendencies[13]. The tendency is the factor we are interested in, the factor predicted by theory. Accidents or disturbances are those factors that produce deviation from the theoretical result[14]. My interpretation is supported by remarks Jevons makes on "The method of means" in his *Principles of Science*. Here, he distinguishes three types of mean, the fictitious mean (a mere representative number serving as convenient mode of comparison), the precise mean result (a result approximately free from disturbances which are known to cancel each other out) and the probable mean result (a result more or less free from unknown and uncertain errors). Jevons says:

> Of these three uses of mean the first is entirely different in nature from the two last, since it does not yield an approximation to any natural quantity, but furnishes us with an arithmetic result comparing the aggregate of certain quantities with their number (Jevons, 1874, p. 360).

Similar views can be found in Jevons's work on the depreciation of gold and on the formation of clouds. It is also interesting in this context to look at what Jevons has to say about "exact science". He contrasts exact science with mathematical science. Exact science is not, as it is for Menger, a matter of the exceptionlessness of laws or of the similarity of types but a matter of correspondence between theory and evidence: the closer the fit between the two, the more exact the science. A mathematical science can be more or less exact, depending on the state of the art of our measuring devices. "Heat" is certainly a quantity capable of intensity. Hence it is to be treated mathematically in the sciences. But it is only after the invention and continuous improvement of thermometers that we can call the branch of physics that deals with temperature exact. All sciences are approximate; exactness is a relative predicate.

I said above that Mengerian economics is a separate science because it treats goods *qua* need-satisfaction capacity and is interested in human action only in so far as it is directed at the satisfaction of needs. Something very similar is going on in Jevons. The "problem of Economics", he writes, is "to satisfy our wants to the utmost with the least effort ... in other words, to maximise pleasure" (Jevons, 1871, p. 40). He then defines a commodity as that "which can afford pleasure or ward off pain" (Jevons, 1871, p. 41), that is, objects are regarded *qua* capacity to afford pleasure. The remainder of his theory follows from these definitions. Jevons's treatment of economics as a separate science is particularly clear in the Introduction to the *Theory of Political Economy*, where he discusses the relation of economics to utilitarianism. Interestingly, he rejects a watered-down version of "pleasure" which would include "any motive which attracts us to a certain conduct" (Jevons, 1871, p. 28). He thinks that motives and feelings are "almost incomparable in power and authority" and that there exists a hierarchy of feelings. However, the science of economics exclusively treats the lowest rank of feelings, and such a separation is possible:

> The calculus of utility aims at supplying the ordinary wants of man at the least cost of labour. Each labourer, in the *absence of other motives*, is supposed to devote his energy to the accumulation of wealth. A higher calculus of moral right and wrong would be needed to show how he may best employ that wealth for the good of others as well as himself. But *when that higher calculus gives no prohibition*, we need the lower calculus to gain us the utmost good in matters of moral indifference (Jevons, 1874, p. 29, author's emphasis).

Aristotle the Platonist versus Aristotle the empiricist

It is clear that Jevons's Aristotelianism differs substantially from Menger's. Jevons was brought up in an empiricist climate and claims that "[a]ll knowledge proceeds originally from experience" (Jevons, 1874, p. 399). In his writings, he stresses issues of observation, measurement, fit between theory and evidence and so on. Menger, on the other hand, often seems wary of the real

world and puts emphasis on "isolation in thought", points out that exact laws should not be tested against empirical data and is aware of the fact that his strict types exist, in part, only in the idea. But they share common metaphysical and epistemological ground, especially as regards the relation between theory and evidence. Both are realists in the sense that they aim at true theories and think that the entities science talks about exist. However, a science has to abstract from concrete detail but the factors one science isolates contribute to a real situation. The world is made up of natural kinds with natural relations between them, and we can find out about these. The reason that Jevons's *Theory of Political Economy* is steeped in mathematics and Menger's is not, in my view, has to do with what they think economics is about and how they conceive what they take it to be about. For Jevons, "[t]he ordinary laws of supply and demand treat entirely of quantities of commodity demanded or supplied, and express the manner in which the quantities vary in connection with the price. In consequence of this fact the laws are mathematical" (Jevons, 1871, p. 4). Utility, the faculty that gives rise to these laws, according to Jevons has only two dimensions, time and intensity, and is thus at least in principle measurable. Hence economics is and must be a mathematical science. Menger too abstracts to a great extent, but the focus of his abstractions is very different. Goods come discretely, as they do in reality. Between production and consumption time passes and thus we are faced with uncertainty. Needs are often qualitatively different from each other, that is, they are incommensurable. These and other of the above listed features of Menger's theory make mathematics a much less natural, and indeed inapplicable, language for economics[15]. Observable prices and quantities of goods demanded and supplied, central features in Jevons's theory, are of only marginal interest for Menger as they are accidental phenomena – almost random manifestations between the boundaries set by value. In his system, it is not possible to use price information to extract something about value.

Schmoller: the philosophy of concrete history

The Historical School's rejection of purely deductive methods and mathematical formalism in particular is well-known. It is very clearly expressed by Wilhelm Roscher:

> ... some scientists [attempted to] fit laws of economics in algebraic formulæ ... But, of course, the advantage of the mathematical mode of expression vanishes the more, the more complex the facts to which they are applied become ... In every description of the life of a nation the algebraic formulæ would become so complicated that they render a continuation of work impossible (Roscher, 1854, p. 67f).

Schmoller's philosophy of economics fits squarely within this tradition. He stresses ideas such as complexity, interdependence of phenomena, context-relativity of facts, processes and development rather than types and their relations. But to conclude from this that Schmoller thinks social science is impossible would be as wrong as characterising him as purely inductivist and his opponents as purely deductivist for both schools of thought employ both

methods. Where they differ is rather in what they take their inductive basis to be. For Menger and Jevons, the basis is about as broad as Descartes's *cogito*. And it is at least in part established by introspection, just like the *cogito*. Jevons calls his method the "complete method", as it "combines observation, deduction, and induction in the most complete and perfect way" (Jevons, 1871, p. 19). This method is common to all empirical sciences. But economics is peculiar in that:

> ... its ultimate laws are known to us immediately by intuition, or, at any rate they are furnished to us ready made by other mental or physical sciences. That every person will choose the greater apparent good; that human wants are more or less quickly satiated; that prolonged labour becomes more and more painful, are a few of the simple inductions on which we can proceed to reason deductively with great confidence (Jevons, 1871, p. 19f).

It is exactly this thinness of the inductive basis Schmoller criticises. In his opinion we have to find out first by a process of observation, classification and discovery of causal relations what the relevant causal factors are and how they are combined to produce the various results. This process cannot be cut short by defining the subject matter of economics in the appropriate way. For Jevons, economics is the science of achieving the greatest pleasure at the smallest pain. Contrast that with Schmoller's characterisation of political economy as "... the system of economic-social processes and activities of the people, which is thought of and acting as a whole, ruled by a unified national spirit and unified material causes ..." (Schmoller, 1923, p. 5)[16]. Economics, as a consequence, is the science that investigates this system. It cannot be thought of as separate from various neighbouring sciences such as ethics, jurisprudence, technology, geography and psychology. For instance, in order to explain certain institutional differences between two societies in their agricultural system, the ideal economist will investigate the respective climatic and geographic conditions, histories, the legislation, the state of technology as well as the motives of the people involved. We can call this type of science polygenetic, as Schmoller thinks that there is a large number of qualitatively different causes (though classifiable into two types: mental and physical) that give rise to the various economic phenomena.

I think it is here where the major difference between Schmoller and the neo-classicals lies. The latter think that economics can be, and perhaps has to be, a separate science: it is possible to isolate one (or a small group of) essential cause(s) and concentrate on the class of effects produced. Schmoller, on the other hand, gives arguments that economics cannot be separate. In his encyclopaedia entry on *Economics and Its Method*, Schmoller states that:

> Certainly, the individual is the starting point of psychological investigations. However, the combination and counter-action of the psychic motives of similar and different people is a thing in itself that cannot be resolved by addition and subtraction of forces (Schmoller, 1904, p. 347).

He then quotes Rümelin in agreement: "The total effect of many individual forces is not a sum or a product as in mechanics" (Schmoller, 1904, p. 347). In

done is highly context dependent. For Menger, the concept of need is far more objective. Certain things cannot be goods because they are cannot satisfy human needs. The examples Menger gives include certain cosmetics, medicines, statues to worship idols, etc.

10. These are besides intensity and duration, certainty, propinquity, fecundity, purity and extent (see Bentham, 1789).
11. In fact, there are three: in the context of feeling, we have time * intensity, in the context of utility, commodity * intensity. That time is continuous is beyond doubt for Jevons.
12. Here, of course, we still have intensity and time as dimensions.
13. The reader who hears John Stuart Mill speaking through Jevons is not mistaken. Jevons accepts Mill's general methodology of economics and discusses his views on pp. 18ff. of the *Theory of Political Economy*.
14. In Aristotelian language, we may call these factors "accidental" versus "essential".
15. In Max Alter's (1990) interpretation the main argument for Menger's rejection of mathematics is his aiming at understanding economic phenomena rather than explaining them (*verstehen* rather than *erklären*). However, I do not see that this is an independent argument. Things could turn out to be mathematical once we have grasped their essence. Therefore, the main point must lie in the nature of need and not in whether we want to understand or explain it.
16. The original reads: "*Denn die Volkswirtschaft is das als ein Ganzes gedachte und wirkende, von dem einheitlichen materiellen Ursachen beherrschte System der wirtschaftlich-gesellschaftlichen Vorgänge und Veranstaltungen des Volkes*".
17. For a more contemporary discussion of this thought, see Georgescu-Roegen (1971, ch. II). Georgescu-Roegen distinguishes between discretely distinct or "arithmomorphic" concepts and dialectical concepts. Arithmomorphic concepts are those whose boundaries are precisely defined, dialectical ones those which have their opposite under their (what Georgescu-Roegen calls) "penumbra", which is something akin to Schmoller's belt of neighbouring meanings. Although I do not like the Hegelian language I tend to agree with Georgescu-Roegen that many, and especially the more fundamental, concepts in economics are of the latter rather than the former type. See also Georgescu-Roegen (1954) for a defence of the claim that the concept of (economic) "want" is not discretely distinct.

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Comment on "Mathematics in economics: Schmoller, Menger and Jevons" by Julian Reiss

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Keywords *Mathematics, Economics*

Abstract *Provides a comment on Reiss' "Mathematics in economics: Schmoller, Menger and Jevons".*

Unfortunately, Julian Reiss fails to notice, that Gustav Schmoller not only criticises "the thinness of the inductive basis" in the contributions of the scientists representing the so-called "deductive methodology" which governed the textbooks and theoretical essays of Carl Menger and William Stanley Jevons and the other utility theorists. Schmoller lay emphasis on the fact that the utility theory of the Benthamite tradition related to pleasure and pain is altogether based on a useless and most questionable psychology. In the first introducing chapters of his comprehensive textbook providing the result of his lifetime research work Schmoller stresses the importance of psychology for political economy but criticises the prevailing use of "utility theories" because these necessarily imply measurability. When visiting the University at Tübingen Schmoller had attended lectures on natural sciences with priority. In his lifetime he had many friends among the most famous scientists of these various disciplines. He remained very interested in questions of methodology throughout his life. His knowledge of the organisation of natural sciences made him complain that introspection as a basis for the utility theories provides information on interpersonally incomparable emotions, which as such cannot be related to an unbiassed, practically defined, provable cardinal metricised scale. This makes, as Schmoller underlines time and again, talking of differences of values, pleasures, pains or utility, of "diminishing marginal utility" or "*ökonomischer Grenznutzen*" (Menger, 1871, pp. 167-8) and also maximization of utility thoroughly meaningless (see Schmoller, 1904, Schmoller's version: "*Gemeinplätze*"; see also Alter, 1990).

The thesis that "political economy pre-supposes an arbitrary definition of man as a being who invariably does that by which he may obtain the greatest amount of necessaries, conveniencies and luxuries with the smallest quantity of labour and physical self-denial" to be appraised method "*a priori*" and "the conclusions correctly deduced from these assumptions would be as true in the abstract as those of mathematics", could not convince Schmoller to bear the capacity to explain any concrete economic proceedings of reality (see Mill, 1948).

So, for Schmoller, making use of Jevons's term "utility" or Menger's "*Bedürfnisbefriedigung*" for mathematical calculations was worthless and of no interest for discovering laws of economical proceedings since no relation could be determined to serve as a sort of provable yardstick for necessary measurement purposes among different beings (Schmoller, 1894; 1904. When writing his article on methodology in 1890, Schmoller arranged for W. Böhmert, a like-minded statistical expert, to submit a paper criticising Jevons's (1891) utility theory – for modern literature see Hempel (1952) and Bohnen (1964)).

Schmoller's textbook on political economy, for this reason, was completely different from any mainstream economics of his time. From the British mathematician William Whewell, who had made use of mathematics to show inconsistencies in Ricardo's and Mill's theories (Whewell, 1830; 1833), and who strongly supported L.A.J. Quetelet's efforts in making use of statistical treatment of economic problems (Whewell held close contact to his friends Quetelet and Richard Jones, and sponsored their invitations to the sessions of the Section for Economics of the British Association for the Advancement of Sciences, which had been founded against strong opposition because of his vigorous engagement (see Hansen, 1968; Backhaus and Hansen, 1998)). Schmoller not only took over the criticism of the utility theory but also his rejection of Mill's Theory of Induction and the preference for statistical treatment of economic reality (Schmoller, 1894, p. 554).

Laws of economic sciences were, according to Schmoller, of a statistical nature, confined to a concrete historical period and institutional equipment, since they normally deal with large numbers of facts. For this reason Schmoller developed statistics to a discipline for the research of regularities in the national economy (Schmoller, 1894, p. 541). In his comprehensive article on methodology of 1892 Schmoller underlined the importance of statistics and the nature of the research of laws in economics. He believed statistical research directed by changing points of interest to be the most important and only basis for all political measures (Schmoller, 1894, p. 559).

Schmoller's textbook therefore is a collection of his research for answers, furnished with statistical information to important economical and political problems acquired during his life (see Schmoller, 1904, Vols I and II).

Schmoller would have made use of any sort of mathematics if to him it had seemed to be productive in answering questions to pressing problems of economic reality. But he could see no improvement in creating illusions by pretending precision.

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The illusion of exactness

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Keywords *Mathematics, Economics, Psychology, Philosophy, History*

Abstract *After having summarized the connections with psychology, philosophy and history, the conclusion of the article is that the application of mathematical models in economics is permissible only in cases when the authors display clearly the time and space limits and other conditions of validity of their models used and, possibly, the attributes of their value-system.*

The following article[1] is written to "audit" the use of mathematical models in economics that – in my opinion – are reliable only within certain limits. Transgressing their competence, they could even become harmful, mirroring the illusion of exactness and expressing, in a disguised manner, the validity of economic laws without any restriction in space and time. My intention was strengthened by the the fact that, in most cases, the authors pointed neither in an explicit nor in an implicit way to the limits and/or the conditions of the methods which they used.

I felt I must not remain silent. Notwithstanding, I knew my paper would open up an embittered debate on its accuracy. As the Latin phrase says: "*Dixi et salvavi animam meam*" ("I spoke and thus I saved my soul").

1. Natural sciences/social sciences

In order to examine the differences better, I would begin by making a comparison taken from physics (mechanics): social scientists are interested in knowing why a certain stone fell to the ground, whence to where, and why precisely to the place in question; whether this event caused further movements and what they were, etc. Natural scientists are only concerned with the general laws of the free fall itself. Toynbee (1956) also distinguishes between the subject matters of natural and social disciplines.

Although the special laws of the social sciences are frequently drawn up in an "ideal/typical" form, they are dependent on concrete space and time. I must also make it clear that their verbal shape sometimes may suggest more of their attributes than any "exact" formula, despite their seemingly inaccurate character. The so-called general laws in social sciences are like the highway code. They contain prescriptions concerning the right way to drive cars but they cannot stipulate where to travel or which would be the right destination, and they do not give definite guidelines regarding speed or which route should be taken.

Homo sapiens has *sui generis* preferences, decision motives that have their sources in the social conditions and the value systems in which, and with which, we live. These confront the prevailing social reality: social sciences are just scrutinizing the political relevances of their forms of motion, looking for optimal solutions of the perceived problems. The decision motives are not

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